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B1. (1) $\int_2^4\left(\int_0^1 x\,dy\right)dx = \int_2^4 x\left(\int_0^1 dy\right)dx = \int_2^4 x\,[y]_0^1\,dx$

$= \int_2^4 x\,dx = \left[\frac{1}{2}x^2\right]_2^4 = \frac{1}{2}(16-4) = 6$ //

(2) $\int_{-3}^2\left(\int_0^1 x^2y\,dx\right)dy = \int_{-3}^2 y\left(\int_0^1 x^2dx\right)dy = \left(\int_0^1 x^2dx\right)\left(\int_{-3}^2 y\,dy\right)$

$= \left[\frac{1}{3}x^3\right]_0^1\cdot\left[\frac{1}{2}y^2\right]_{-3}^2 = \frac{1}{3}\cdot\frac{1}{2}(2^2-(-3)^2) = -\frac{5}{6}$ //

(3) $\int_0^1\left(\int_{y^2}^y y\,dx\right)dy = \int_0^1 y\,[x]_{y^2}^y\,dy = \int_0^1 y(y-y^2)\,dy$

$= \left[\frac{1}{3}y^3 - \frac{1}{4}y^4\right]_0^1 = \frac{1}{3}-\frac{1}{4} = \frac{1}{12}$ //

(4) $\int_0^1\left(\int_0^x \sqrt{x+y}\,dy\right)dx = \int_0^1\left[\frac{1}{\frac{1}{2}+1}(x+y)^{\frac{1}{2}+1}\right]_0^x dx = \int_0^1\left[\frac{2}{3}(x+y)^{\frac{3}{2}}\right]_0^x dx$

$= \frac{2}{3}\int_0^1\left((2x)^{\frac{3}{2}} - x^{\frac{3}{2}}\right)dx = \frac{2}{3}(2\sqrt{2}-1)\int_0^1 x^{\frac{3}{2}}dx = \frac{2}{3}(2\sqrt{2}-1)\left[\frac{1}{\frac{3}{2}+1}x^{\frac{3}{2}+1}\right]_0^1$

$= \frac{2}{3}\cdot(2\sqrt{2}-1)\cdot\frac{2}{5} = \frac{4}{15}(2\sqrt{2}-1)$ //

(5) $\int_0^2\left(\int_{\frac{y}{2}}^y e^{x+y}\,dx\right)dy = \int_0^2 e^y\left(\int_{y/2}^y e^x\,dx\right)dy$

$= \int_0^2 e^y\,[e^x]_{y/2}^y\,dy = \int_0^2 e^y(e^y - e^{y/2})\,dy = \int_0^2(e^{2y} - e^{3y/2})\,dy$

$= \left[\frac{1}{2}e^{2y} - \frac{2}{3}e^{3y/2}\right]_0^2 = \frac{1}{2}e^4 - \frac{2}{3}e^3 - \frac{1}{2} + \frac{2}{3} = \frac{e^4}{2} - \frac{2e^3}{3} + \frac{1}{6}$ //

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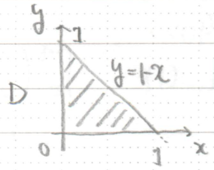
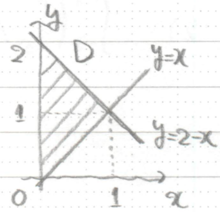
B2

(1) $D = \{(x, y) : 0 \leq x \leq 1,\ 0 \leq y \leq 1-x\}$ より



$$\iint_D x\,dxdy = \int_0^1 \left(\int_0^{1-x} x\,dy \right) dx = \int_0^1 x \left(\int_0^{1-x} dy \right) dx$$

$$= \int_0^1 x [y]_0^{1-x} dx = \int_0^1 (x - x^2) dx = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \,/\!/$$



(2) $D = \{(x, y) : 0 \leq x \leq 1,\ x \leq y \leq 2-x\}$ より

$$\iint_D xy\,dxdy = \int_0^1 \left(\int_x^{2-x} xy\,dy \right) dx$$

$$= \int_0^1 x \left(\int_x^{2-x} y\,dy \right) dx = \int_0^1 x \left[\frac{1}{2} y^2 \right]_x^{2-x} dx$$

$$= \int_0^1 x \left(\frac{1}{2}(2-x)^2 - \frac{1}{2}x^2 \right) dx = 2\int_0^1 x(1-x)\,dx = 2\left(\frac{1}{2} - \frac{1}{3} \right) = \frac{1}{3} \,/\!/$$

(3) $$\iint_D \frac{x}{y^2}\,dxdy = \int_1^2 \left(\int_1^{x^2} \frac{x}{y^2}\,dy \right) dx = \int_1^2 x \left(\int_1^{x^2} y^{-2}\,dy \right) dx$$

$$= \int_1^2 x \left[\frac{1}{-2+1} y^{-2+1} \right]_1^{x^2} dx = \int_1^2 x [-y^{-1}]_1^{x^2} dx = \int_1^2 x(-x^{-2} + 1)\,dx$$

$$= \int_1^2 \left(x - \frac{1}{x} \right) dx = \left[\frac{1}{2}x^2 - \log x \right]_1^2 = \frac{1}{2}(4-1) - \log 2 + \log 1 = \frac{3}{2} - \log 2 \,/\!/$$

(4) $$\iint_D x\,dxdy = \int_0^{\pi} \left(\int_0^{\sin x} x\,dy \right) dx = \int_0^{\pi} x [y]_0^{\sin x} dx$$

$$= \int_0^{\pi} x \sin x\,dx = [-x\cos x]_0^{\pi} + \int_0^{\pi} \cos x\,dx = \pi + [\sin x]_0^{\pi} = \pi \,/\!/$$

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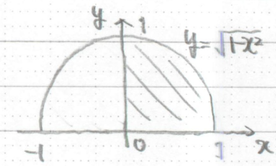
(5) $D = \{(x,y) : 0 \leq x \leq 1,\ 0 \leq y \leq \sqrt{1-x^2}\}$ より



$$\iint_D \sqrt{1-x^2}\,dxdy = \int_0^1 \left(\int_0^{\sqrt{1-x^2}} \sqrt{1-x^2}\,dy \right) dx$$

$$= \int_0^1 \sqrt{1-x^2}\,\left[y \right]_0^{\sqrt{1-x^2}} dx = \int_0^1 (1-x^2)dx = 1 - \frac{1}{3} = \frac{2}{3} //$$

B3 (1) $D = \{(x,y) : -3 \leq x \leq 0,\ 0 \leq y \leq -x\}$

$= \{(x,y) : 0 \leq y \leq 3,\ -3 \leq x \leq -y\}$ より

$$\iint_D f(x,y)dxdy = \int_{-3}^0 \left(\int_0^{-x} f(x,y)dy \right) dx = \int_0^3 \left(\int_{-3}^{-y} f(x,y)dx \right) dy //$$

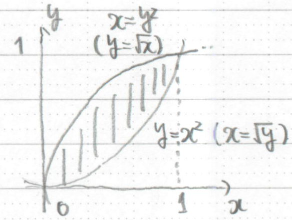
(2) $D = \{(x,y) : 0 \leq x \leq 1,\ 0 \leq y \leq \sqrt{1-x^2}\}$

$= \{(x,y) : 0 \leq y \leq 1,\ 0 \leq x \leq \sqrt{1-y^2}\}$ より

$$\iint_D f(x,y)dxdy = \int_0^1 \left(\int_0^{\sqrt{1-x^2}} f(x,y)dy \right) dx = \int_0^1 \left(\int_0^{\sqrt{1-y^2}} f(x,y)dx \right) dy //$$

(3) $D = \{(x,y) : 0 \leq x \leq 1,\ x^2 \leq y \leq \sqrt{x}\}$

$= \{(x,y) : 0 \leq y \leq 1,\ y^2 \leq x \leq \sqrt{y}\}$ より



$$\iint_D f(x,y)dxdy = \int_0^1 \left(\int_{x^2}^{\sqrt{x}} f(x,y)dy \right) dx$$

$$= \int_0^1 \left(\int_{y^2}^{\sqrt{y}} f(x,y)dx \right) dy //$$