Date

P.35

B1 (1) $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n} = \lim_{m\to-\infty}\left(1+\frac{1}{m}\right)^{-m}$ ($m=-n$ と置く $\overset{n\to\infty}{\Leftrightarrow} m\to-\infty$)

$= \lim_{m\to-\infty}\frac{1}{\left(1+\frac{1}{m}\right)^{m}} = \frac{1}{\lim_{m\to-\infty}\left(1+\frac{1}{m}\right)^{m}} = \frac{1}{e}$ // (∵ p.33 (7.2))

(2) $\lim_{x\to0}\frac{\log(1+x)}{x+x^{2}} = \lim_{x\to0}\frac{\log(1+x)}{(1+x)x} = \lim_{x\to0}\frac{1}{1+x}\cdot\frac{\log(1+x)}{x}$

$= \lim_{x\to0}\frac{1}{1+x}\cdot\lim_{x\to0}\frac{\log(1+x)}{x} = \frac{1}{1+0}\cdot1 = 1$ // (∵ p.34 (7.4))

(3) $\lim_{h\to0}\frac{1-e^{ah}}{h+ah^{2}} = \lim_{h\to0}\frac{-a}{1+ah}\cdot\frac{e^{ah}-1}{ah} = \lim_{h\to0}\frac{-a}{1+ah}\cdot\lim_{h\to0}\frac{e^{ah}-1}{ah}$

$= \frac{-a}{1+0}\cdot\lim_{k\to0}\frac{e^{k}-1}{k} = -a\cdot1 = -a$ // ($k=ah$ と置く $\overset{h\to0}{\Leftrightarrow} k\to0$ (∵ $a\neq0$))

(4) $\lim_{x\to0}\frac{e^{x}-e^{-x}}{x} = \lim_{x\to0}\frac{e^{x}-1+1-e^{-x}}{x} = \lim_{x\to0}\left(\frac{e^{x}-1}{x}+\frac{e^{-x}-1}{-x}\right)$

$= \lim_{x\to0}\frac{e^{x}-1}{x} + \lim_{x\to0}\frac{e^{-x}-1}{-x} = 1 + \lim_{y\to0}\frac{e^{y}-1}{y} = 1+1=2$ // ($y=-x$ と置く $\overset{x\to0}{\Leftrightarrow} y\to0$)

※ 次の4種の極限は既知として利用してもよい：

$\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x} = e$ (7.2) $\lim_{x\to0}(1+x)^{\frac{1}{x}} = e$ (7.3)

$\lim_{x\to0}\frac{\log(1+x)}{x} = 1$ (7.4) $\lim_{x\to0}\frac{e^{x}-1}{x} = 1$ (7.5) □

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p.39

B1. (1) $y' = (e^{2x})' = (2x)' e^{2x} = 2e^{2x}$ //

(2) $y' = (\log(4x+1))' = \dfrac{(4x+1)'}{4x+1} = \dfrac{4}{4x+1}$ //

(3) $y' = (\sqrt{x})' e^{\sqrt{x}} = \dfrac{1}{2\sqrt{x}} e^{\sqrt{x}}$ // $\left((\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \right)$

(4) $y' = \dfrac{(x+\frac{1}{x})'}{x+\frac{1}{x}} = \dfrac{1-\frac{1}{x^2}}{x+\frac{1}{x}} = \dfrac{(1-\frac{1}{x^2})\times x^2}{(x+\frac{1}{x})\times x^2} = \dfrac{x^2-1}{(x^2+1)x}$ //

(5) $y' = (5xe^{-3x})' = 5(xe^{-3x})'$

$= 5\{x'(e^{-3x}) + x\cdot(e^{-3x})'\} = 5(e^{-3x} + x\cdot(-3)e^{-3x}) = 5(1-3x)e^{-3x}$ //

(6) $y' = (e^x)'\cdot\log x + e^x(\log x)'$

$= e^x\cdot\log x + e^x\cdot\dfrac{1}{x} = e^x\left(\log x + \dfrac{1}{x}\right)$ //

(7) $y' = \dfrac{(\log x)'}{\log x} = \dfrac{\frac{1}{x}}{\log x} = \dfrac{1}{x\log x}$ //

(8) $y' = \left(\log\sqrt{\dfrac{x-a}{x+a}}\right)' = \left(\log\left(\dfrac{x-a}{x+a}\right)^{\frac{1}{2}}\right)' = \left(\dfrac{1}{2}\log\dfrac{x-a}{x+a}\right)'$

$= \dfrac{1}{2}(\log(x-a) - \log(x+a))' = \dfrac{1}{2}\cdot\left(\dfrac{1}{x-a} - \dfrac{1}{x+a}\right)$

$= \dfrac{1}{2}\,\dfrac{x+a-(x-a)}{(x-a)(x+a)} = \dfrac{a}{x^2-a^2}$ //

※ 合成関数の微分の公式より次が導かれる:

$(e^{f(x)})' = f'(x)e^{f(x)}, \quad (\log f(x))' = \dfrac{f'(x)}{f(x)}$ □

Date

B2 (1) 両辺の対数をとると $\log y = \log 2^{-x} = -x\log 2$. この両辺を$x$で微分して

$$(\log y)' = \frac{y'}{y},\quad (-x\log 2)' = -\log 2 \quad \therefore \frac{y'}{y} = -\log 2 \quad \therefore y' = -2^{-x}\log 2$$

(2) 両辺の対数をとると

$$\log y = \log\sqrt{(x+1)(x+2)(x+3)} = \log\left((x+1)(x+2)(x+3)\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\log(x+1) + \log(x+2) + \log(x+3)\right)$$

この両辺をxで微分して

$$\frac{y'}{y} = \frac{1}{2}\left(\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3}\right) \quad \therefore y' = \frac{1}{2}\sqrt{(x+1)(x+2)(x+3)}\left(\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3}\right)$$

(3) 両辺の対数をとると

$$\log y = \log\frac{(x-1)^3(x-3)^5}{\sqrt{x-2}} = 3\log(x-1) + 5\log(x-3) - \frac{1}{2}\log(x-2)$$

この両辺をxで微分して

$$\frac{y'}{y} = \frac{3}{x-1} + \frac{5}{x-3} - \frac{1}{2(x-2)} = \frac{6(x-2)(x-3) + 10(x-1)(x-2) - (x-1)(x-3)}{2(x-1)(x-2)(x-3)}$$

$$= \frac{15x^2 - 56x + 53}{2(x-1)(x-2)(x-3)}$$

$$\therefore y' = \frac{(x-1)^2(x-3)^4}{2\sqrt{(x-2)^3}}(15x^2 - 56x + 53)$$

(4) 両辺の対数をとると $\log y = \log x^{-x} = -x\log x$ この両辺をxで微分して

$$\frac{y'}{y} = (-x)'\log x + (-x)\cdot(\log x)' = -\log x - 1 \quad \therefore y' = -x^{-x}(\log x + 1)$$

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P.45

1. $\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $\cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$$\tan\left(-\frac{\pi}{3}\right) = \frac{\sin(-\pi/3)}{\cos(-\pi/3)} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$$

$$\sec\left(-\frac{\pi}{3}\right) = \frac{1}{\cos(-\pi/3)} = \frac{1}{\frac{1}{2}} = 2.$$

$$\operatorname{cosec}\left(-\frac{\pi}{3}\right) = \frac{1}{\sin(-\pi/3)} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}}$$

$$\cot\left(-\frac{\pi}{3}\right) = \frac{1}{\tan(-\pi/3)} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

P.45

2. (1) $\sin(x+h) - \sin x = \sin x \cos h + \cos x \sin h - \sin x$ (∵ 加法定理)

$= \sin x \cdot \cos 2\cdot\frac{h}{2} + \cos x \cdot \sin 2\cdot\frac{h}{2} - \sin x$

$= \sin x \cdot \left(1 - 2\sin^2\frac{h}{2} - 1\right) + \cos x \cdot 2 \cdot \sin\frac{h}{2}\cos\frac{h}{2}$ (∵ 倍角の公式).

$= 2\sin\frac{h}{2}\left(-\sin x \cdot \sin\frac{h}{2} + \cos x \cdot \cos\frac{h}{2}\right) = 2\sin\frac{h}{2} \cdot \cos\left(x + \frac{h}{2}\right)$ // (∵ 加法定理)

(2) $\cos(x+h) - \cos x = \cos x \cdot \cos h - \sin x \cdot \sin h - \cos x$

$= \cos x \cdot \left(1 - 2\sin^2\frac{h}{2} - 1\right) - \sin x \cdot 2\sin\frac{h}{2} \cdot \cos\frac{h}{2}$

$= -2\sin\frac{h}{2} \cdot \left(\cos x \cdot \sin\frac{h}{2} + \sin x \cdot \cos\frac{h}{2}\right) = -2\sin\frac{h}{2} \cdot \sin\left(x + \frac{h}{2}\right)$ //

※ P.43 の (9.15), (9.16) を既知とすれば

$$\sin(x+h) - \sin x = 2\cos\frac{x+h+x}{2} \cdot \sin\frac{x+h-x}{2} = 2\cos\left(x + \frac{h}{2}\right)\sin\frac{h}{2}$$

と計算できる。

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P49 1

(1) $a \neq 0$ とする。このとき

$$(与式) = \lim_{x\to 0} \frac{\sin ax}{ax} \cdot \frac{1}{\frac{\sin bx}{bx}} \cdot \frac{ax}{bx} = \frac{a}{b} \lim_{x\to 0} \frac{\sin ax}{ax} \cdot \frac{1}{\lim_{x\to 0} \frac{\sin bx}{bx}} = \frac{a}{b}$$ //

これは $a=0$ のときも含めて成立する。

(2) $$(与式) = \lim_{x\to 0} x \cdot \frac{\sin x^2}{x^2} = \lim_{x\to 0} x \cdot \lim_{x\to 0} \frac{\sin x^2}{x^2}$$

$t = x^2$ と置けば $x \to 0 \Leftrightarrow t \to 0+0$ より $\displaystyle\lim_{x\to 0} \frac{\sin x^2}{x^2} = \lim_{t\to 0+0} \frac{\sin t}{t} = 1$

$\therefore$ $(与式) = 0 \times 1 = 0$ //

P49 2

(1)〜(3) は簡単なので省略。

(4) $y' = 2\cdot 2\sin x \cdot \cos x - 2\cdot \cos x \cdot (-\sin x) = 3 \cdot 2\sin x \cos x = 3\sin 2x$ //

(別解) $y = 2\cdot \frac{1-\cos 2x}{2} - \frac{1+\cos 2x}{2} = \frac{1}{2} - \frac{3}{2}\cos 2x$ $\quad\therefore\ y' = -\frac{3}{2}(-\sin 2x)\cdot 2 = 3\sin 2x$ //

(5) $y' = -\dfrac{\cos x}{\sin^2 x} = -\cot x \cdot \operatorname{cosec} x$ // (どちらでもよい)

(6) $$y' = \frac{\cos x(\sin x + \cos x) - \sin x(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x + \cos^2 x + 2\sin x \cdot \cos x} = \frac{1}{1+\sin 2x}$$ //

(7) $y' = \dfrac{1}{\cos^2\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{1}{2\sqrt{x}\cos^2\sqrt{x}} = \dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}$ // (どちらでもよい)

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P.49 2 (つづき)

(8) $y' = 2\sec x \cdot (\sec x)' = 2 \cdot \sec x \cdot (\tan x \cdot \sec x) = 2\tan x \cdot \sec^2 x$ //

(9) $y' = \dfrac{1}{(\sqrt{1+\cos^2 x})^2} \cdot \left\{ \cos x \cdot \sqrt{1+\cos^2 x} - \sin x \cdot \dfrac{1}{2\sqrt{1+\cos^2 x}} \cdot 2\cos x \cdot (-\sin x) \right\}$

$= \dfrac{\cos x}{\sqrt{(1+\cos^2 x)^3}} \cdot (1+\cos^2 x + \sin^2 x) = \dfrac{2\cos x}{\sqrt{(1+\cos^2 x)^3}}$ //

(10) $y' = -\dfrac{1}{\sin^2 x} + \dfrac{1}{2} \cdot 2\tan x \cdot \dfrac{1}{\cos^2 x} = -\mathrm{cosec}^2 x + \tan x \cdot \sec^2 x$ //

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P.54 1

(1) $y = \sin^{-1} t$, $t = 2x$ とする.

$$y' = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{1}{\sqrt{1-t^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$ //

(2) $$y' = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot (\sqrt{x})' + \frac{1}{1+(2\sqrt{x})^2} \cdot (2\sqrt{x})'$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} + \frac{1}{1+4x} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}} \left(\frac{1}{\sqrt{1-x}} + \frac{2}{1+4x} \right)$$ //

(3) $$y' = -\frac{1}{x^2} \tan^{-1} x + \frac{1}{x} \cdot \frac{1}{1+x^2} = \frac{1}{x^2} \left(\frac{x}{1+x^2} - \tan^{-1} x \right)$$ //

(4) $$y' = \sin^{-1} \frac{x}{2} + x \cdot \frac{1}{\sqrt{1-(\frac{x}{2})^2}} \cdot \frac{1}{2} = \sin^{-1} \frac{x}{2} + \frac{x}{\sqrt{4-x^2}}$$ //

p.58

B1. (1) $y' = 5(3x-2)^4 \cdot 3 = 15(3x-2)^4$

$y'' = 15 \cdot 4(3x-2)^3 \cdot 3 = 180(3x-2)^3$ //

(2) $y = (x^2+2)^{\frac{1}{2}}$　　$y' = \frac{1}{2}(x^2+2)^{-\frac{1}{2}} \cdot 2x = x(x^2+2)^{-\frac{1}{2}}$

$y'' = (x^2+2)^{-\frac{1}{2}} + x \cdot \left(-\frac{1}{2}(x^2+2)^{-\frac{3}{2}} \cdot 2x\right)$

$= (x^2+2)^{-\frac{1}{2}} - x^2(x^2+2)^{-\frac{3}{2}} = (x^2+2)^{-\frac{3}{2}}(x^2+2-x^2) = \frac{2}{\sqrt{(x^2+2)^3}}$ //

(3) $y' = \frac{1}{1+x^2}$.　$y'' = -\frac{2x}{(1+x^2)^2}$ //

B2. (1) $y = (x+2)^{-1}$　$y' = -(x+2)^{-2}$,　$y'' = (-1)(-2)(x+2)^{-3}$

$\cdots$ $y^{(n)} = (-1)(-2)\cdots(-n)(x+2)^{-n-1} = \frac{(-1)^n n!}{(x+2)^{n+1}}$ //

(2) $y = \frac{A}{x+1} + \frac{B}{x-1}$ と分解されたとすると

$y = \frac{Ax-A+Bx+B}{(x-1)(x+1)} = \frac{(A+B)x+B-A}{(x-1)(x+1)} = \frac{1}{(x-1)(x+1)}$

分子を比較して　$A+B=0$.　$B-A=1$　$\therefore A=-\frac{1}{2}$, $B=\frac{1}{2}$

$\therefore y = -\frac{1}{2}(x+1)^{-1} + \frac{1}{2}(x+1)^{-1}$

$y' = -\frac{1}{2}(-1)(x+1)^{-2} + \frac{1}{2}(-1)(x+1)^{-2}$,　$y'' = -\frac{1}{2}(-1)(-2)(x+1)^{-3} + \frac{1}{2}(-1)(-2)(x+1)^{-3}$

$\cdots$ $y^{(n)} = -\frac{1}{2}(-1)(-2)\cdots(-n)(x+1)^{-n-1} + \frac{1}{2}(-1)(-2)\cdots(-n)(x-1)^{-n-1}$

$= \frac{(-1)^n n!}{2}\left\{\frac{1}{(x-1)^{n+1}} - \frac{1}{(x+1)^{n+1}}\right\}$ //

(3) $y' = 2e^{2x+3}$ $\quad y'' = 2^2 e^{2x+3}$, $\quad \cdots \quad y^{(n)} = 2^n \cdot e^{2x+3}$ //

(4) $y' = 3\cos(3x+1) = 3\sin(3x+1+\frac{\pi}{2})$ $\qquad (\sin(\theta+\frac{\pi}{2}) = \cos\theta)$

$y'' = 3^2\cos(3x+1+\frac{\pi}{2}) = 3^2\sin(3x+1+\frac{\pi}{2}+\frac{\pi}{2})$

$\cdots\ y^{(n)} = 3^n\sin(3x+1+\underbrace{\frac{\pi}{2}+\cdots+\frac{\pi}{2}}_{n\text{個}}) = 3^n\sin(3x+1+\frac{n}{2}\pi)$ //

(5) $y = \log\sqrt{4x-3} = \log(4x-3)^{\frac{1}{2}} = \frac{1}{2}\log(4x-3)$ $\qquad ((\log f(x))' = \frac{f'(x)}{f(x)})$

$y' = \frac{1}{2}(4x-3)^{-1}\cdot 4 = \frac{4}{2}(4x-3)^{-1}$

$y'' = \frac{4}{2}(-1)(4x-3)^{-2}\cdot 4 = \frac{4^2}{2}\cdot(-1)(4x-3)^{-2}$

$y''' = \frac{4^2}{2}\cdot(-1)(-2)(4x-3)^{-3}\cdot 4 = \frac{4^3}{2}(-1)\cdot(-2)(4x-3)^{-3}$

$\cdots\ y^{(n)} = \frac{4^n}{2}(-1)(-2)\cdots(-(n-1))(4x-3)^{-n} = \frac{(-1)^{n-1}\cdot 4^n\cdot(n-1)!}{2(4x-3)^n}$ //

(6) $y' = a^x\log a$ $\quad y'' = a^x(\log a)^2$, $\quad \cdots\ .\ y^{(n)} = a^x(\log a)^n$ //