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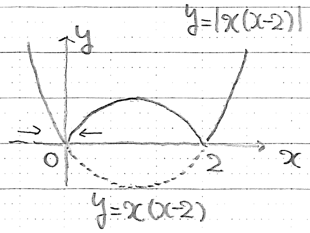

B.1 (1) $x<0$ のとき $f(x)=x(x-2)$, $x>0$ のとき $f(x)=-x(x-2)$ だから

$$\lim_{h\to 0-0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0-0}\frac{h(h-2)}{h}=\lim_{h\to 0-0}(h-2)=-2$$

$$\lim_{h\to 0+0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0+0}\frac{-h(h-2)}{h}=-\lim_{h\to 0+0}(h-2)=-(-2)=2$$

右極限と左極限は一致せず. したがって極限 $\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$ は存在しない. よって $f(x)$ は $x=0$ で微分可能ではない

(2) $$\lim_{h\to 0-0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0-0}\frac{|(0+h)^3|-|0^3|}{h}=\lim_{h\to 0-0}\frac{-h^3}{h}=0$$

$$\lim_{h\to 0+0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0+0}\frac{|(0+h)^3|-|0^3|}{h}=\lim_{h\to 0+0}\frac{h^3}{h}=0$$

右極限 = 左極限 = 0, したがって $\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=0$ となり. よって $f(x)$ は $x=0$ で微分可能である.

B.3 (1) $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2+(x+h)-(x^2+x)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2+\cancel{x}+h-\cancel{x^2}-\cancel{x}}{h}=\lim_{h\to 0}(2x+1+h)=2x+1 \,/\!/$$

(2) $$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$

$$=\lim_{h\to 0}\frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}} \,/\!/$$

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(3) $f'(x) = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right) = \lim_{h\to 0} \frac{1}{h} \frac{x-(x+h)}{(x+h)x} = \lim_{h\to 0} \frac{-1}{(x+h)x} = -\frac{1}{x^2}$ //

(4) $f'(x) = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right) = \lim_{h\to 0} \frac{1}{h} \frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x+h}\sqrt{x}}$

$= \lim_{h\to 0} \frac{1}{h} \frac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{\sqrt{x+h}\sqrt{x}(\sqrt{x}+\sqrt{x+h})} = \lim_{h\to 0} \frac{1}{h} \cdot \frac{x-(x+h)}{\sqrt{x+h}\sqrt{x}(\sqrt{x}+\sqrt{x+h})}$

$= \lim_{h\to 0} \frac{-1}{\sqrt{x+h}\sqrt{x}(\sqrt{x}+\sqrt{x+h})} = -\frac{1}{\sqrt{x}\cdot\sqrt{x}(\sqrt{x}+\sqrt{x})} = -\frac{1}{2x\sqrt{x}}$ //

※ B2 は B3で $x=1$ とした場合なので省略する.

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B.1 (1) $y' = (x^3-4x^2+3x-2)' = (x^3)'-(4x^2)'+(3x)'-(2)'$

$= (x^3)'-4(x^2)+3(x)'-2\cdot(1)'$

$= 3x^2-4\cdot 2x+3\cdot 1-2\cdot 0 = 3x^2-8x+3$ //

(2) $y' = ((2x+3)^3)' = 3(2x+3)^{3-1}\cdot(2x+3)' = 3(2x+3)^2\cdot 2 = 6(2x+3)^2$ //

(別解) $y' = (8x^3+3\cdot 4x^2\cdot 3+3\cdot 2x\cdot 3^2+3^3)'$

$= 8(x^3)'+36(x^2)'+54(x)'+27\cdot(1)'$

$= 8\cdot 3x^2+36\cdot 2x+54\cdot 1+0 = 24x^2+72x+54$ //

(3) $y' = 3\cdot(x^2+2x-1)^{3-1}\cdot(x^2+2x-1)' = 3(x^2+2x-1)^2\cdot(2x+2) = 6(x^2+2x-1)^2(x+1)$ //

(4) $y' = \dfrac{(3x)'\cdot(2x^2+1)-(3x)\cdot(2x^2+1)'}{(2x^2+1)^2} = \dfrac{3\cdot(2x^2+1)-3x\cdot(4x)}{(2x^2+1)^2} = \dfrac{3(1-2x^2)}{(2x^2+1)^2}$ //

(5) $y' = (x^{\frac{3}{4}})' = \dfrac{3}{4}x^{\frac{3}{4}-1} = \dfrac{3}{4}x^{-\frac{1}{4}} = \dfrac{3}{4}(\sqrt[4]{x})^{-1} = \dfrac{3}{4\sqrt[4]{x}}$ //

(6) $y' = ((x+1)^{-\frac{2}{3}})' = -\dfrac{2}{3}(x+1)^{-\frac{2}{3}-1} = -\dfrac{2}{3}(x+1)^{-\frac{5}{3}} = -\dfrac{2}{3\sqrt[3]{(x+1)^5}}$ //

(7) $y' = -\dfrac{(1+\sqrt{x})'}{(1+\sqrt{x})^2} = -\dfrac{(x^{\frac{1}{2}})'}{(1+\sqrt{x})^2} = -\dfrac{\frac{1}{2}\cdot x^{-\frac{1}{2}}}{(1+\sqrt{x})^2} = -\dfrac{\frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2} = -\dfrac{1}{2\sqrt{x}(1+\sqrt{x})^2}$ //

(8) $y' = \left(((x+1)(x-2))^{\frac{1}{2}}\right)' = \dfrac{1}{2}((x+1)(x-2))^{-\frac{1}{2}}\cdot((x+1)(x-2))'$

$= \dfrac{1}{2}(\sqrt{(x+1)(x-2)})^{-1}\cdot\{(x+1)'(x-2)+(x+1)(x-2)'\}$

$= \dfrac{1}{2\sqrt{(x+1)(x-2)}}(x-2+x+1) = \dfrac{2x-1}{2\sqrt{(x+1)(x-2)}}$ //

(別解) $y' = ((x+1)^{\frac{1}{2}}\cdot(x-2)^{\frac{1}{2}})' = ((x+1)^{\frac{1}{2}})'\cdot(x-2)^{\frac{1}{2}}+(x+1)^{\frac{1}{2}}((x-2)^{\frac{1}{2}})'$

$= \dfrac{1}{2}(x+1)^{-\frac{1}{2}}\cdot(x-2)^{\frac{1}{2}}+\dfrac{1}{2}(x+1)^{\frac{1}{2}}(x-2)^{-\frac{1}{2}} = \dfrac{1}{2}\left(\dfrac{\sqrt{x-2}}{\sqrt{x+1}}+\dfrac{\sqrt{x+1}}{\sqrt{x-2}}\right)$

$= \dfrac{1}{2}\dfrac{(\sqrt{x-2})^2+(\sqrt{x+1})^2}{\sqrt{x+1}\sqrt{x-2}} = \dfrac{x-2+x+1}{2\sqrt{(x+1)(x-2)}} = \dfrac{2x-1}{2\sqrt{(x+1)(x-2)}}$ //

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(9) $y' = \dfrac{x'\sqrt{x^2+1} - x\cdot(\sqrt{x^2+1})'}{(\sqrt{x^2+1})^2} = \dfrac{\sqrt{x^2+1} - x\cdot\dfrac{x}{\sqrt{x^2+1}}}{x^2+1}$

$$\left(\begin{aligned}(\sqrt{x^2+1})' &= ((x^2+1)^{\frac{1}{2}})' \\ &= \frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot(x^2+1)' \\ &= \frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x = \frac{x}{\sqrt{x^2+1}}\end{aligned}\right)$$

$= \dfrac{\left(\sqrt{x^2+1} - \dfrac{x^2}{\sqrt{x^2+1}}\right)\times\sqrt{x^2+1}}{(x^2+1)\quad\times\sqrt{x^2+1}} = \dfrac{x^2+1-x^2}{(x^2+1)\sqrt{x^2+1}}$

$= \dfrac{1}{(x^2+1)\sqrt{x^2+1}}$ // または $\dfrac{1}{\sqrt{(x^2+1)^3}}$

(10) $y' = \left(\left(\dfrac{1-x}{1+x}\right)^{\frac{1}{2}}\right)' = \dfrac{1}{2}\left(\dfrac{1-x}{1+x}\right)^{-\frac{1}{2}}\cdot\left(\dfrac{1-x}{1+x}\right)' = \dfrac{1}{2}\left(\dfrac{1+x}{1-x}\right)^{\frac{1}{2}}\cdot\dfrac{(1-x)'(1+x)-(1-x)(1+x)'}{(1+x)^2}$

$= \dfrac{1}{2}\,\dfrac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}\cdot\dfrac{-2}{(1+x)^2} = -\dfrac{1}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{3}{2}}} = -\dfrac{1}{\sqrt{(1-x)(1+x)^3}}$ //

(別解) $y' = \left(\dfrac{(1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}}}\right)' = \dfrac{((1-x)^{\frac{1}{2}})'\cdot(1+x)^{\frac{1}{2}} - (1-x)^{\frac{1}{2}}((1+x)^{\frac{1}{2}})'}{((1+x)^{\frac{1}{2}})^2}$

$= \dfrac{\frac{1}{2}(1-x)^{-\frac{1}{2}}\cdot(-1)\cdot(1+x)^{\frac{1}{2}} - (1-x)^{\frac{1}{2}}\cdot\frac{1}{2}(1+x)^{-\frac{1}{2}}}{1+x} = -\dfrac{1}{2}\cdot\dfrac{(1-x)^{-\frac{1}{2}}(1+x)^{\frac{1}{2}} + (1-x)^{\frac{1}{2}}\cdot(1+x)^{-\frac{1}{2}}}{1+x}$

$= -\dfrac{1}{2}\,\dfrac{(1-x)^{-\frac{1}{2}}(1+x)^{-\frac{1}{2}}(1+x) + (1-x)^{-\frac{1}{2}}(1-x)\cdot(1+x)^{-\frac{1}{2}}}{1+x}$

$= -\dfrac{1}{2}\,\dfrac{1+x+1-x}{(1-x)^{\frac{1}{2}}\cdot(1+x)^{\frac{1}{2}}(1+x)} = -\dfrac{1}{\sqrt{(1-x)(1+x)^3}}$ //

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B1. (1) $y=\sqrt{1-x}$ は $x=1-y^2$ $(y \geqq 0)$ と同値だから、逆関数定理より

$$\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}=\frac{1}{(1-y^2)'}=\frac{1}{-2y}=-\frac{1}{2\sqrt{1-x}} \quad (1-x>0)$$ //

(2) $y=\sqrt[3]{x}$ は $x=y^3$ と同値だから、逆関数定理より

$$\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}=\frac{1}{3y^2}=\frac{1}{3(\sqrt[3]{x})^2}=\frac{1}{3\sqrt[3]{x^2}} \quad (x \neq 0)$$ //

B2. (1) $$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{(3t+2)'}{(2t-1)'}=\frac{3}{2}$$ //

(2) $$\frac{dx}{dt}=\frac{(1-t^2)'(1+t^2)-(1-t^2)(1+t^2)'}{(1+t^2)^2}=\frac{-2t(1+t^2)-(1-t^2)\cdot 2t}{(1+t^2)^2}=-\frac{4t}{(1+t^2)^2}$$

$$\frac{dy}{dt}=\frac{(2t)'(1+t^2)-2t(1+t^2)'}{(1+t^2)^2}=\frac{2(1+t^2)-2t\cdot(2t)}{(1+t^2)^2}=\frac{2(1-t^2)}{(1+t^2)^2}$$

$$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2(1-t^2)/(1+t^2)^2}{-4t/(1+t^2)^2}=-\frac{1-t^2}{2t} \quad (t \neq 0)$$ //

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